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Photoelastic constants of ADP

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Though a number of authors have reported the photoelastic behaviour of ADP, there are discrepancies in the literature with regard to its photoelastic constants. In the present study all the static and dynamic data have been obtained again by repeating the experiments and the most acceptable values of the stress-optical and strain-optical constants are calculated by the method of least squares. The results are discussed in the light of earlier studies.

1. Introduction

Ammonium dihydrogen phosphate (ADP) is a uniaxial crystal of the tetragonal system belonging to D_{2d} class in Schoenflies notation. It has seven non-vanishing, independent stress-optical (q_{11}, q_{12}, q_{13}, q_{31}, q_{33}, q_{44} and q_{66}) and strain-optical (p_{11}, p_{12}, p_{13}, p_{31}, p_{33}, p_{44} and p_{66}) constants both in Pockels' and Bhagavantam's schemes of photoelastic constants. ADP has been subjected to photoelastic studies by a number of investigators including West and Makas [1], Willard [2], Carpenter [3], Devoit [4], Vlokh and Lutsiv-Shumskii [5] and Achyuthan and Breazeale [6]. Of these, the first five have employed only static methods to obtain a few of the stress-optical constants. Achyuthan and Breazeale have employed an ultrasonic method to obtain the strain-optical ratios and they combined this information with the static data of Carpenter and obtained q_{13} and q_{33} for the first time. Furthermore, they have calculated the strain optical constants p_{11}, p_{12}, p_{13}, p_{31} and p_{33}. Later, Dixon [7] obtained only the numerical magnitudes of some of the p_{ij} using the technique of Bragg diffraction of light from an acoustic wave train. Ziauddin and Narasimhamurty [8] have determined all the q_{ij} and p_{ij} by repeating all the static and dynamic observations. Recently Davis and Vedam [9] reported the effect of hydrostatic pressure on the refractive indices μ_0 and μ_e of ADP by an interferometric method. They found that both ordinary and extraordinary refractive indices of ADP increase with pressure, contradicting the conclusions that could be drawn from the earlier observations of Achyuthan and Breazeale, and Ziauddin and Narasimhamurty. Therefore the present investigations have been undertaken to check up the values of all the stress-optical and strain-optical constants.

2. Experimental procedure and results

For this purpose, a new technique developed by the authors [10] has been employed to obtain the Brewster's constants C_λ for various orientations and the ultrasonic method due to Narasimhamurty [11] to determine the strain-optical ratios p_{ij}/p_{kl}. The axes of the crystal blanks† were identified by their morphology and later confirmed by optical and X-ray methods. In the present investigations, C_λ was determined for seven orientations and the strain-optical ratios for three orientations. Apart from this data, we have two observations from the hydrostatic data of Davis and Vedam. Now, the most acceptable values of the stress-optical constants are calculated by the least-squares method by combin-

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TABLE I

Serial no.	Direction of Stress	Direction of Observation	Expression for C_λ	$C_\lambda \times 10^{13}$ (cm^2 dyne^{-1}) at $\lambda = 5890$ Å	Remarks
1	[100] or [010]	[010] or [100]	$\frac{1}{2}(n_x^3 q_{11} - n_z^3 q_{31})$	4.298	Serial nos. 1 to 7 are by the authors
2	[100] or [010]	[001]	$\frac{1}{2}n_x^3 (q_{11} - q_{12})$	1.248	
3	[001]	[100] or [010]	$\frac{1}{2}(n_z^3 q_{33} - n_x^3 q_{13})$	3.455	
4	M	M′	$\frac{1}{8}n_{yz}^3 (q_{11}+ q_{13}+ q_{31}+ q_{33}+ 2q_{44}) - \frac{1}{4} n_x^3 (q_{12}+ q_{13})$	−5.276	
5	L or L′	[001]	$\frac{1}{2} n_x^3 q_{66}$	−27.35	
6	[100]	M or M′	$\frac{1}{2}n_x^3 q_{11} - \frac{1}{4}n_{yz}^3 (q_{12}+ q_{31})$	2.994	
7	L	L′	$\frac{1}{4}n_x^3 (q_{11}+ q_{12}+ q_{66}) - 2n_z^3 q_{31}$	−8.883	
8			$(q_{11}+ q_{12}+ q_{13})$	8.020	Serial nos. 8 and 9 are by Davis and Vedam
9			$(2q_{31}+ q_{33})$	8.782	

Note—M is a direction equally inclined to [010] and [001] in the yz plane.
L is a direction equally inclined to [100] and [010] in the xy plane.
M′ and L′ are perpendicular to M and L respectively in the yz plane and xy plane.

ing all this data. The expressions for C_λ (after correcting for thickness change) along with those obtained from hydrostatic data are collected in Table I.

For ADP, only one of the ratios, namely p_{11}/p_{31} (Table II) has been found to be definitely more than unity and for the other two ratios it has not been possible to rely on their values since they are close to unity. Here it must be emphasized that when the ratio of the strain-optical constants is nearer to unity, one should precisely know whether the magnitude is greater or smaller than unity. As will be clear from the discussion, this particular mistake can change the entire set of photoelastic constants both in magnitude and sign. Hence the equations in Table I are combined with the ratio p_{11}/p_{31} determined by one of the authors (Pettersen) by more sensitive ultrasonic methods. The values of q_{ij} thus obtained at 5890 Å and at room temperature (21 °C) are collected in Table IIIa, along with those reported by some of the earlier authors. The strain-optical constants are now calculated from these values of q_{ij} using the well-known relations

$$p_{ij} = \sum_{j=1}^{6} q_{hj}\, C_{jl},\ (h, l = 1 \text{ to } 6)$$

and are given in Table IIIb. The refractive indices used in the calculations are taken from Zernike [12] and the elastic constants from Hearmon [13].

At the first sight of the equations in Table I, one may feel that a knowledge of the strain-optical ratios determined by the ultrasonic method is not essential since there are seven expressions due to stress birefringence studies and two due to hydrostatic data, while the unknown constants are only seven. But these nine equations together cannot be solved since some of the equations are not independent. For example, the two equations for path retardations under hydrostatic pressure are not independent of the expressions at serial numbers 1, 2 and 3 in Table I since any one of them can be deduced from the remaining four equations. Hence it was

TABLE II

Serial no.	Direction of Excitation	Direction of Observation	Expression for the ratio R	Ratio R
1	[100]	[010]	p_{11}/p_{31}	1.62
2	[100]	[001]	p_{11}/p_{12}	1.12
3	[001]	[100]	p_{33}/p_{13}	1.01

TABLE IIIa

$q_{ij} \times 10^{13}$ cm^2 dyne^{-1}	Carpenter	Achyuthan and Breazeale	Ziauddin and Narasimhamurty	Present
q_{11}	8.6	—	7.62	4.40 ± 0.34
q_{12}	7.9	—	6.80	3.71 ± 0.32
q_{13}	12.3	—	11.94	0.34 ± 0.11
q_{31}	—	−37.3	−33.47	2.02 ± 0.28
q_{33}	—	−35.7	−34.87	2.65 ± 0.50
q_{44}	−5.8	—	−6.15	−6.70 ± 0.37
q_{66}	−12.2	—	−16.50	−15.25 ± 0.16

TABLE IIIb

p_{ij}	Achyuthan and Breazeale	Ziauddin and Narasimhamurty	Dixon	Present
p_{11}	−0.11	−0.11	0.302	0.319
p_{12}	−0.15	−0.16	0.246	0.277
p_{13}	−0.93	−0.84	0.236	0.169
p_{31}	0.20	0.18	0.195	0.197
p_{33}	−0.71	−0.70	0.263	0.167
p_{44}	—	−0.056	—	−0.058
p_{66}	—	−0.099	—	−0.091

Note—Dixon gives only numerical values of p_{ij}, since his method cannot yield the sign.

necessary to take at least one strain-optical ratio.

3. Discussion

From the present study it has been found that a wrong sign has been attributed by Carpenter [14] for the expressions at serial numbers 1 and 6 in Table I. It has also been noticed that Achyuthan and Breazeale [6], and Ziauddin and Narasimhamurty [8] have used the reciprocal of the actual ratios in their calculations and thus the photoelastic constants reported by them could not satisfy the observations on the change in refractive indices due to hydrostatic pressure. The probable reason for their mistake may be the wrong determination of the axes of the crystals or confusion between the polarization direction and vibration direction of light. Furthermore, Achyuthan and Breazeale have assigned a negative sign to the ratio p_{11}/p_{31}, which has been found to be erroneous as discussed below. In order to test our own observations, the following procedure was adopted.

The stress birefringence data (serial numbers 1 to 7 in Table I) obtained in the present study is combined with the positive value of p_{11}/p_{31}, the magnitude being the same as that obtained presently. The q_{ij} thus obtained are found to satisfy the hydrostatic data in sign as well as in magnitude. But if we put a negative sign or reciprocal of the magnitude as was used by earlier authors, it was found that not only the results do not satisfy the observations of hydrostatic pressure studies, but also give absurd values for the other two strain-optical ratios. This clearly indicates that the sign of p_{11}/p_{31} is positive and the order of magnitude obtained presently is satisfactory. Furthermore, if the stress birefringence data at serial numbers 2 to 7 in Table I is combined with the ratio p_{11}/p_{31} as obtained presently, the expression at serial number 1 in Table I is found to have a positive sign. Thus the present observations are put to a critical test from all aspects and are found to be satisfactory.

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